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# MEASUREMENT OF THE VELOCITY OF LIGHT BETWEEN MOUNT WILSON AND MOUNT SAN ANTONIO

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ABSTRACT

The following is a continuation of the experiments described in the *Astrophysical Journal*, 60, 256, 1924. The arrangement of apparatus differs slightly from that of the former investigation, allowing a more nearly normal incidence on the facets of the revolving mirror, and providing greater symmetry as well as increase in illumination.

Five independent series of measurements made with different revolving mirrors (one of steel having the form of a prism with eight facets, and another with twelve, and three of glass with eight, twelve, and sixteen facets) gave results showing a remarkable agreement.

The final result for the velocity of light *in vacuo* is 299,796 km per second.

The increasing appreciation of the importance of this fundamental constant of nature and the anticipation of the possibility of attaining a considerable increase in accuracy in its measurement justified the acceptance of the generous invitation of Dr. G. E. Hale, then director of the Mount Wilson Observatory, to make use of the facilities there available. It is with sincere appreciation that I take this opportunity of extending my acknowledgment of the many courtesies received and of the cordial co-operation of Dr. Hale and subsequently of Dr. Adams and the entire staff of the Observatory.

The result of the preliminary experiments described in a previous article[1] gave for the final value of the velocity of light in air 299,735 km per second.

[1] *Astrophysical Journal*, 60, 256, 1924.

The correction[1] to *vacuum* is given by

$$n-1=(n_0-1)\frac{d}{d_0}=(n_0-1)\frac{p}{p_0}\frac{T_0}{T}$$

$$\frac{p}{p_0}=.822$$

and

$$\frac{T_0}{T}=.932\ ,$$

whence

$$n-1=.000294\times .765$$

or

$$n=1.000225\ ,$$

giving 67 km for the correction.[2]

Accordingly, for the preliminary determination

$$V=299{,}735+67$$

or

$$V=299{,}802\ .$$

A second series of observations with the glass octagon was begun in July of 1925, the arrangement of apparatus being essentially the same as in the preliminary work, except that instead of driving the electric fork ($N=132$) by make and break between platinum points, the fork ($N=528$) was driven by a vacuum-tube circuit, giving a rate far more nearly constant.

This rate was measured by comparison with a free pendulum furnished by the United States Coast and Geodetic Survey, as in the previous work; but with an important improvement in the stroboscopic method. This consists in allowing the light from a very narrow slit to fall on a mirror attached to the pendulum, forming, by means of a good achromatic lens, an image of the slit in the plane of an edge of the fork, where it is observed by an eyepiece.

[1] The correction should be applied to the individual observations; but the result is not appreciably altered by taking the mean values given above.

[2] This was erroneously given as 85 km in the article cited.

This method of making the stroboscopic comparison was found to be far more convenient and accurate than that described in the work of last year.

As before, the octagonal mirror making 528 turns per second rotates through one-eighth of a turn during the time of passage of light from the revolving mirror to the distant station and return, thus presenting the succeeding face of the mirror to the returning beam at (very nearly) the same angle as at rest.

The observations consist, then, in increasing the speed of the revolving mirror until the stroboscopic image between fork and mirror is stationary, at which instant the measurement is taken of the small angle $\alpha_1$ by which the displacement of the image differs from 90°. The direction of rotation is then reversed, and a similar series of observations furnishes the angle $\alpha_2$. If $\alpha=\alpha_1+\alpha_2$, then the angle through which the mirror rotates during the time required for light to travel through the distance $2D$ will be $\pi/4-\alpha/4$, and the velocity is given by $V=(16ND)/(1-\alpha/\pi)$.

If $1/n$ is the period of the (optical) beats between the fork and the pendulum and $1/\nu$ that of the coincidences between the C.G.S. pendulum and the true seconds, and if $N$ is the nearest whole number (in the present instance 528), then, as both $\alpha$ and $\nu$ are small,

$$V=\frac{16D}{1-\nu}(N+n)\left(1+\frac{\alpha}{\pi}\right),$$

or, since $\alpha$ and $n$ are small,

$$V=\frac{16\times 35425.15\times 528}{1-.00051}\left(1+\frac{\alpha}{\pi}+\frac{n}{N}\right)\qquad D=35425.1$$

$$V=299{,}425\left(1+\frac{\alpha}{\pi}+\frac{n}{N}\right).$$

Table I gives the results of ten series of observations with the glass octagon.

These results, as well as those previously published,[1] are to be regarded as preliminary. The definitive measurements were begun in June, 1926, and continued until the middle of September.

[1] *Astrophysical Journal*, **60**, 256, 1924.

TABLE I

TEN SERIES OF OBSERVATIONS

| | $a/\pi$ | $n/N$ | $V_a$ |
|---|---|---|---|
| I | 0.00077 | 0.00013 | 299,695 |
| II | .00057 | .00015 | 299,651 |
| III | .00044 | .00038 | 299,671 |
| IV | .00037 | .00047 | 299,677 |
| V | .00054 | .00045 | 299,722 |
| VI | .00047 | .00043 | 299,695 |
| VII | .00032 | .00068 | 299,725 |
| VIII | .00017 | .00070 | 299,686 |
| IX | .00018 | .00076 | 299,707 |
| X | 0.00021 | 0.00058 | 299,662 |
| Mean velocity in air | | | 299,689 |
| Correction | | | +67 |
| *V in vacuo* | | | 299,756 |

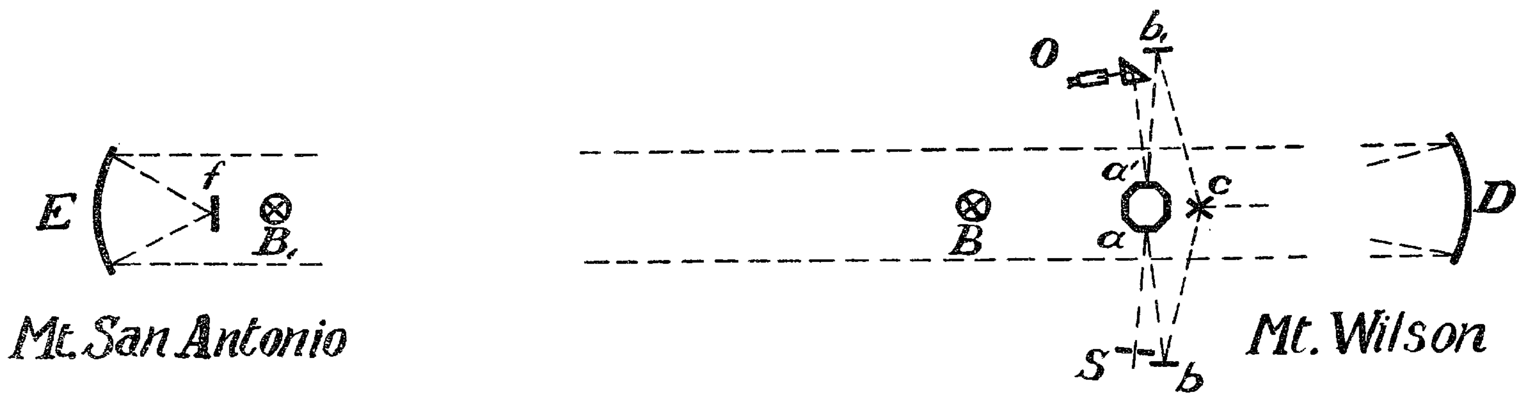


FIG. 1.—Arrangement of apparatus

The arrangement of apparatus at the home station was slightly different, as shown in Figure 1, the advantage being an increase in intensity in consequence of greater effective width of the light beam falling on the revolving mirror at nearly normal incidence, as well as greater symmetry.

With this layout a series of observations with the same small glass octagon gave the results recorded in Table II. The numbers

TABLE II

| | *a* | *b* | *c* | *V* | Wt. |
|---|---|---|---|---|---|
| I | 0.00059 | 0.00028 | 0.00072 | 299,747 | 2 |
| II | .00046 | .00040 | .00073 | 299,747 | 2 |
| III | .00045 | .00038 | .00073 | 299,738 | 3 |
| IV | .00057 | .00036 | .00072 | 299,762 | 3 |
| V | .00047 | .00033 | .00073 | 299,729 | 3 |
| VI | .00052 | .00038 | .00073 | 299,759 | 3 |
| VII | .00061 | .00041 | .00073 | 299,792 | 1 |
| VIII | .00048 | .00038 | .00072 | 299,744 | 4 |
| IX | .00049 | .00036 | .00072 | 299,741 | 4 |
| X | .00047 | .00040 | .00072 | 299,747 | 4 |
| XI | .00044 | .00042 | .00072 | 299,744 | 4 |
| XII | 0.00042 | 0.00042 | 0.00073 | 299,741 | 4 |
| Weighted mean | | | | 299,746 | |
| Correction | | | | +67 | |
| Velocity *in vacuo* | | | | 299,813 | |

given are the means of three series of observations, each series containing six (double) observations.

On account of the small values of $a = \alpha/\pi$, $b = n/N$, and $c = \nu$, the formula for the velocity may be written

$$V = 16DN(1 + a + b + c) .$$

With $D = 35{,}425$ and $N = 528$, this gives

$$V = 299270 + V(a + b + c) .$$

The results with the glass octagon are shown in Table II.

Giving these three results the weights 1, 2, and 5, respectively, the weighted mean of all the observations with the glass octagon is $V = 299{,}797$.

DEFINITIVE MEASUREMENTS

Toward the close of the work of the summer of 1925 an attempt was made to attain a speed of rotation of 528 turns per second with a glass octagon nearly twice as large as that used in the experiments just described. Owing probably to some defect in the glass, it burst at a speed of 400, thus terminating the work. In order to provide against a repetition of such an accident four mirrors were constructed. Two of these were of glass, of about the same construction, but twice as large as the small octagon, a photograph of which is given in Figure 2. The first of these had twelve facets and the second sixteen, the diameters being $6\frac{1}{4}$ and $7\frac{1}{2}$ cm, respectively, and the speed of rotations 350 and 264. The flat end of the steel axle on which the glass mirrors are mounted rests on a steel flat with a small aperture for oil, under pressure a trifle less than that which would lift it from its bearing. The driving power is furnished by an air blast issuing through two nozzles and impinging on the vanes of a paddle wheel.

Regulation of the speed was obtained by a counterblast controlled by a valve by means of which the speed could be kept constant for several seconds (as indicated by the immobility of the stroboscopic image), quite sufficient for the measurement of the corresponding small displacement of the image.

The other two mirrors were of nickel steel and furnished by Mr. Elmer A. Sperry. A photograph of the first, an octagon, is shown in Figure 3.

The driving power is furnished by an air blast from four nozzles impinging on the buckets attached to the axle.[1] The flat end of the latter rests on a single ball-bearing.

All four mirrors gave excellent results notwithstanding the fact that time was insufficient for the attainment of very great accuracy in the surfaces. This was particularly noticeable in the steel mirrors, the lack of homogeneity making the process of polishing and figuring much more difficult than in the case of the glass mirrors.[2]

[1] The mirrors actually employed were provided with duplicate bucket wheels and nozzles, allowing of reversal of the direction of rotation.

[2] All are now in process of refiguring in preparation for a resumption of operations during the winter, when atmospheric conditions are likely to be more favorable.

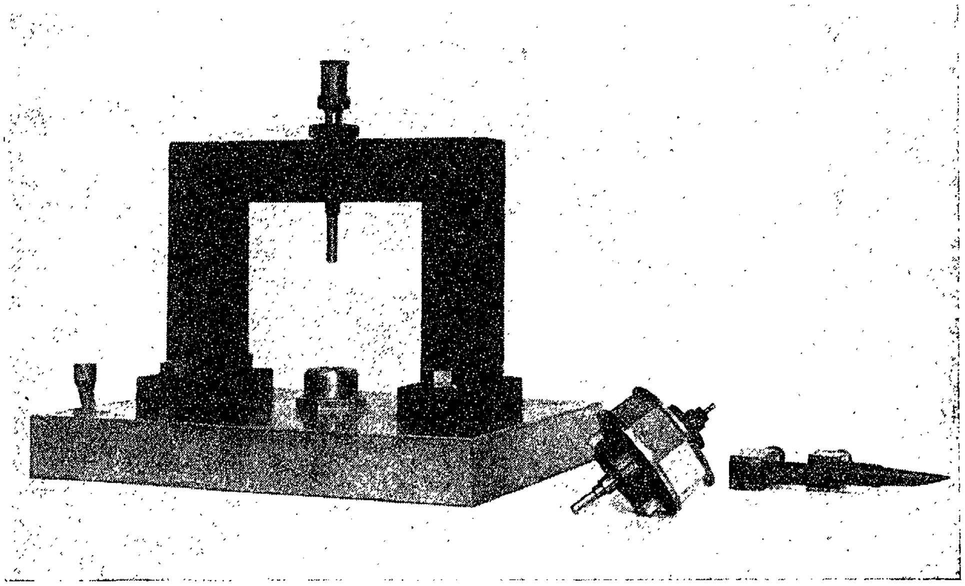

FIG. 2a.—The small octagon detached from its mounting

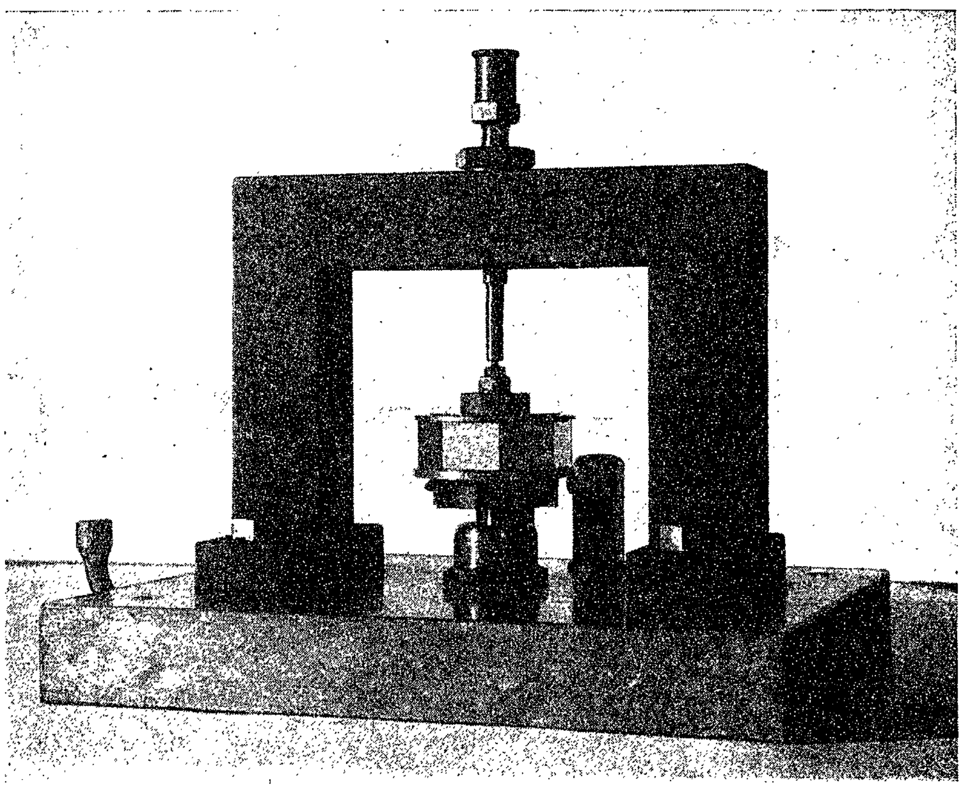

FIG. 2b.—The small octagon in its mounting

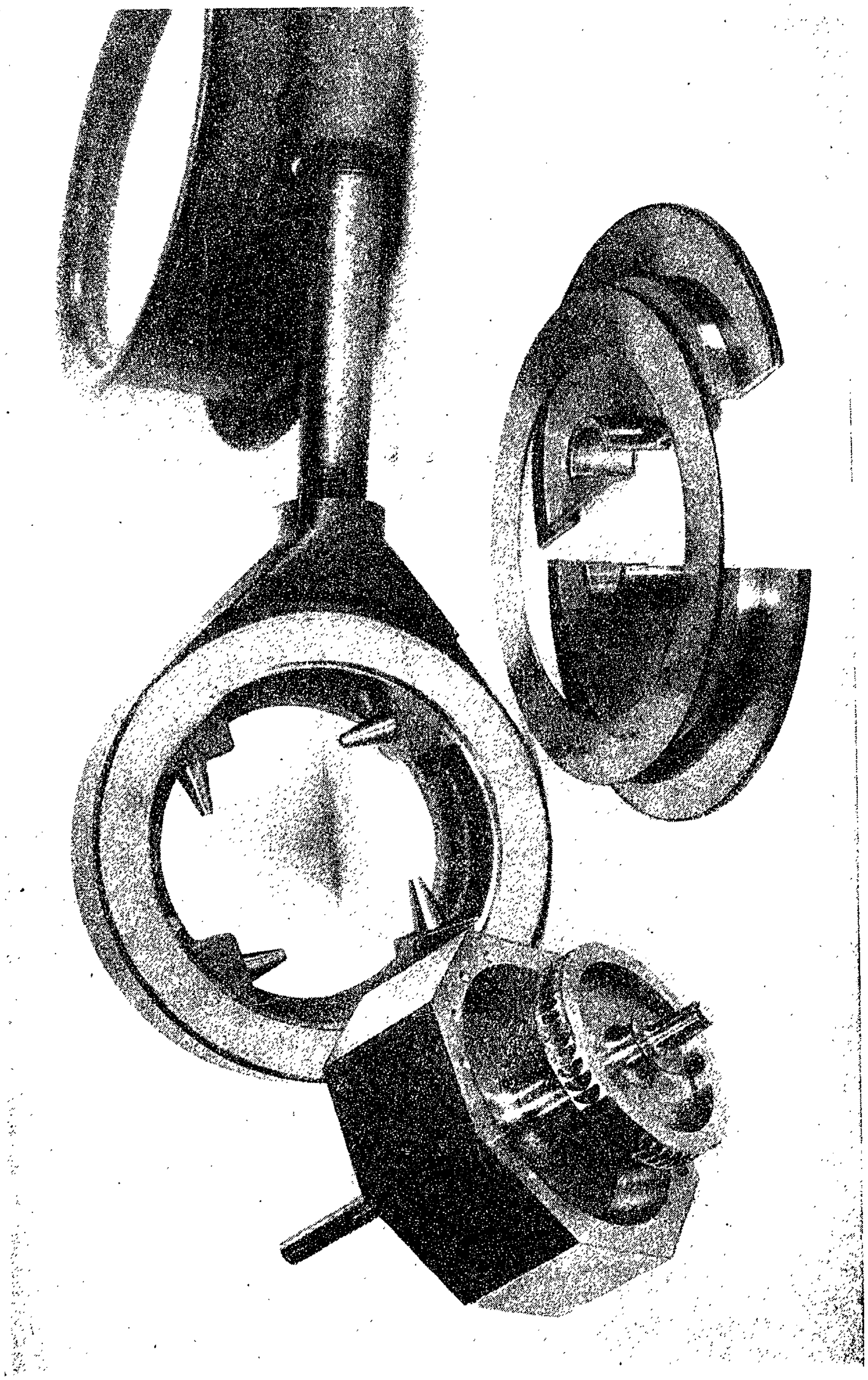

FIG. 3.—The octagon of nickel steel furnished by Mr. Sperry

With a twelve-facet glass mirror and the layout of apparatus and method of procedure essentially the same as in last year's work, the following results were obtained:

$$V=\frac{24DN}{1-\nu}(N+n)\left(1+\frac{3\alpha}{2\pi}\right)$$

or if

$$\frac{3\alpha}{2\pi}=a\quad \frac{n}{N}=b$$

and

$$-\nu=c\qquad D=35424.5$$
$$r=53\text{ cm}$$
$$V=299{,}265+3(a+b+c)\ .$$

The results obtained are shown in Table III.

TABLE III

TWELVE-FACET GLASS MIRROR

| | $a$ | $b$ | $c$ | $VS$ | $V$ | Wt. |
|---|---|---|---|---|---|---|
| I | −0.00018 | 0.00100 | 0.00075 | 471 | 299,736 | 1 |
| II | .00047 | .00040 | .00073 | 480 | 299,745 | 3 |
| III | .00058 | .00026 | .00073 | 471 | 299,733 | 3 |
| IV | .00022 | .00061 | .00072 | 465 | 299,730 | 3 |
| V | .00012 | .00062 | .00071 | 435 | 299,700 | 1 |
| VI | − .00007 | .00088 | .00073 | 462 | 299,727 | 5 |
| VII | .00020 | .00098 | .00073 | 453 | 299,718 | 5 |
| VIII | .00004 | .00085 | .00073 | 462 | 299,727 | 5 |
| IX | .00009 | .00100 | .00073 | 492 | 299,757 | 1 |
| X | − .00021 | .00114 | .00074 | 501 | 299,766 | 2 |
| XI | .00050 | .00037 | .00074 | 483 | 299,748 | 2 |
| XII | .00071 | .00009 | .00073 | 459 | 299,724 | 5 |
| XIII | .00052 | .00034 | .00073 | 477 | 299,742 | 5 |
| XIV | .00003 | .00073 | .00075 | 453 | 299,718 | 5 |
| XV | 0.00004 | 0.00071 | 0.00075 | 450 | 299,715 | 5 |
| Weighted mean | | | | | 299,729 | |
| Correction | | | | | +67 | |
| Velocity *in vacuo* | | | | | 299,796 | |

A series of measurements with the sixteen-facet glass mirror, the formula for which is

$$V=\frac{32DN}{1+2\,\alpha/\pi}\,,\qquad D=35424.5$$
$$\gamma=55\text{ cm}$$

gave the results shown in Table IV.

TABLE IV

| | *a* | *b* | *c* | *VS* | *V* | Wt. |
|---|---|---|---|---|---|---|
| $\{I_{I}$ | 0.00159 | −0.00089 | 0.00076 | 438 | 299,703 | ..... |
| $\{I_{II}$ | .00115 | − .00033 | .00076 | 474 | 299,739 | |
| I | $(2\times I_{II}+I_{I})/3$ | .......... | .......... | ..... | 299,727 | 1 |
| II | .00051 | .00045 | .00071 | 501 | 299,766 | 2 |
| III | .00038 | .00052 | .00071 | 483 | 299,748 | 2 |
| $\{IV_{I}$ | .00006 | .00073 | .00071 | 450 | 299,715 | ..... |
| $\{IV_{II}$ | .00079 | − .00006 | .00073 | 438 | 299,703 | |
| IV | $(2\times IV_{II}+IV_{I})/3$ | .......... | .......... | ..... | 299,707 | 5 |
| V | .00090 | − .00009 | .00073 | 462 | 299,727 | 5 |
| $\{VI_{I}$ | .00111 | − .00029 | .00072 | 462 | 299,727 | ..... |
| $\{VI_{II}$ | .00074 | .00013 | .00072 | 477 | 299,742 | |
| VI | $(2\times VI_{II}+VI_{I})/3$ | .......... | .......... | ..... | 299,737 | 4 |
| VII | .00085 | .00011 | .00072 | 504 | 299,769 | 3 |
| VIII | .00097 | − .00013 | .00069 | 459 | 299,724 | 2 |
| IX | .00104 | .00008 | .00070 | 498 | 299,763 | 2 |
| X | .00081 | .00002 | .00071 | 450 | 299,715 | 4 |
| XI | .00091 | .00007 | .00071 | 465 | 299,730 | 4 |
| XII | .00112 | .00027 | .00069 | 462 | 299,727 | 2 |
| XIII | .00117 | .00027 | .00069 | 477 | 299,742 | 2 |
| XIV | .00098 | .00009 | .00070 | 477 | 299,742 | 3 |
| XV | 0.00104 | −0.00009 | 0.00070 | 495 | 299,760 | 3 |
| Weighted mean | .............. | .......... | .......... | ..... | 299,736 | ..... |
| Correction | .............. | .......... | .......... | ..... | +67 | ..... |
| Velocity *in vacuo* | .............. | .......... | .......... | ..... | 299,803 | ..... |

A second series with the sixteen-facet glass mirror gave the results summarized in Table V.

TABLE V

| | *a* | *b* | *c* | *VS* | *V* | Wt |
|---|---|---|---|---|---|---|
| I | 0.00067 | 0.00029 | 0.00071 | 501 | 299,766 | 1 |
| II | .00019 | .00060 | .00073 | 456 | 299,721 | 5 |
| III | .00020 | .00061 | .00073 | 462 | 299,727 | 5 |
| IV | .00053 | .00029 | .00074 | 468 | 299,733 | 1 |
| V | .00039 | .00034 | .00075 | 444 | 299,709 | 5 |
| VI | .00049 | .00029 | .00075 | 459 | 299,724 | 5 |
| VII | .00066 | .00009 | .00073 | 444 | 299,709 | 2 |
| VIII | .00064 | .00010 | .00073 | 441 | 299,706 | 2 |
| IX | .00078 | .00006 | .00074 | 474 | 299,739 | 3 |
| X | .00102 | − .00015 | .00072 | 477 | 299,742 | 3 |
| XI | .00053 | .00025 | .00073 | 453 | 299,718 | 3 |
| XII | .00059 | .00018 | .00072 | 447 | 299,712 | 1 |
| XIII | 0.00071 | 0.00023 | 0.00072 | 498 | 299,763 | 1 |
| Weighted mean | ......... | .......... | ......... | ....... | 299,722 | ....... |
| Correction | ......... | .......... | ......... | ....... | +67 | ....... |
| Velocity *in vacuo* | ......... | .......... | ......... | ....... | 299,789 | ....... |

A series of measurements with the twelve-facet steel mirror gave the results shown in Table VI.

TABLE VI

| | *a* | *b* | *c* | *VS* | *V* | Wt. |
|---|---|---|---|---|---|---|
| $I_{I}$ | 0.00063 | 0.00016 | 0.00072 | 453 | 299,718 | .... |
| $I_{II}$ | .00030 | .00046 | .00072 | 444 | 299,709 | |
| I | $(2 \times I_{II} + I_{I})/3$ | ........ | ........ | .... | 299,712 | 2 |
| II | .00040 | .00043 | .00072 | 465 | 299,730 | 4 |
| III | .00051 | .00032 | .00072 | 465 | 299,730 | 4 |
| IV | .00052 | .00030 | .00072 | 462 | 299,727 | 5 |
| V | .00052 | .00031 | .00072 | 465 | 299,730 | 5 |
| VI | .00055 | .00031 | .00072 | 474 | 299,739 | 5 |
| VII | .00001 | .00078 | .00072 | 453 | 299.718 | 2 |
| VIII | .00054 | .00027 | .00073 | 462 | 299,727 | 2 |
| IX | .00056 | .000315 | .000736 | 483 | 299,748 | 3 |
| X | .00052 | .00027 | .00074 | 459 | 299,724 | 3 |
| XI | 0.00054 | 0.00023 | 0.00074 | 453 | 299,718 | 3 |
| Weighted mean | .............. | ........ | ........ | .... | 299,729 | .... |
| Correction | .............. | ........ | ........ | .... | +67 | .... |
| Velocity *in vacuo* | .............. | ........ | ........ | .... | 299,796 | .... |

Table VII gives the results obtained with the steel octagon.

TABLE VII

| | *a* | *b* | *c* | *VS* | *V* | Wt. |
|---|---|---|---|---|---|---|
| I | 0.00027 | 0.00057 | 0.00071 | 465 | 299,730 | 3 |
| II | .00032 | .00049 | .00071 | 456 | 299,721 | 3 |
| III | .00059 | .00028 | .00069 | 468 | 299,733 | 3 |
| IV | .00057 | .00025 | .00069 | 453 | 299,718 | 5 |
| $V_{I}$ | .00008 | .00072 | .00072 | 456 | 299,712 | |
| $V_{II}$ | .00054 | .00030 | .00072 | 468 | 299,733 | .... |
| $V_{III}$ | .00076 | .00005 | .00072 | 459 | 299,724 | |
| V | Mean of 3 | ........ | ........ | .... | 299,723 | 3 |
| $VI_{I}$ | .00065 | .00027 | .00072 | 492 | 299,757 | 3 |
| $VI_{II}$ | .00081 | .00005 | .00072 | 474 | 299,739 | .... |
| VI | $(2 \times VI_{II} + VI_{I})/3$ | ........ | ........ | .... | 299,744 | 3 |
| VII | .00035 | .00049 | .00072 | 468 | 299,733 | 3 |
| VIII | .00059 | .00024 | .00072 | 465 | 299,730 | 5 |
| IX | .00055 | .00026 | .00072 | 459 | 299,724 | 5 |
| X | .00056 | .00025 | .00072 | 459 | 299,724 | 5 |
| XI | 0.00058 | 0.00025 | 0.00072 | 465 | 299,730 | 5 |
| Weighted mean | .............. | ........ | ........ | .... | 299,728 | .... |
| Correction | .............. | ........ | ........ | .... | +67 | .... |
| Velocity *in vacuo* | .............. | ........ | ........ | .... | 299,795 | .... |

These results are collected in Table VIII.

TABLE VIII

| Mirror | Year | *N* | *n* | *V* | Wt. |
|---|---|---|---|---|---|
| Glass 8 | 1925 | 528 | 150 | 299,802 | 1 |
| Glass 8 | 1925 | 528 | 200 | 299,756 | 1 |
| Glass 8 | 1926 | 528 | 216 | 299,813 | 3 |
| Steel 8 | 1926 | 528 | 195 | 299,795 | 5 |
| Glass 12 | 1926 | 352 | 270 | 299,796 | 3 |
| Steel 12 | 1926 | 352 | 218 | 299,796 | 5 |
| Glass 16 | 1926 | 264 | 270 | 299,803 | 5 |
| Glass 16 | 1926 | 264 | 234 | 299,789 | 5 |
| Weighted mean | .......... | .......... | .......... | 299,796±4 | .......... |

When grouped in series of observations with the five mirrors the results show a much more striking agreement, as follows:

| | |
|---|---|
| Glass 8 | 299,797 |
| Steel 8 | 299,795 |
| Glass 12 | 299,796 |
| Steel 12 | 299,796 |
| Glass 16 | 299,796 |

The ready success of the measurements at the distance of twenty-two miles, the majority of which were made under conditions not the most favorable (haze and smoke from forest fires; imperfections of surfaces of the revolving mirrors), would seem to indicate the feasibility of a measurement at a considerably greater distance. A convenient location for the distant mirror (diameter, 40 in.) was found at Mount San Jacinto, eighty-two miles distant. In a preliminary trial, light was returned, but so enfeebled by smoke that measurements were quite impracticable.

It is hoped that during the winter season the rains will clear the atmosphere sufficiently for accurate measurement, and accordingly work will begin again toward the end of December, 1926.

I take this opportunity to express my appreciation of the generous gift of $10,000 by Mr. Martin A. Ryerson which made this investigation possible.

## APPENDIX I

Measurement of the length $2D$ of the light-path.

The light-path as shown in Figure 1 is from the surface $a$ of the revolving mirror to $b\ c\ D\ E\ f\ E\ D\ c\ b_1\ a_1$.

The measurements of the separate elements are as follows:

Distance between bench marks $B\ B_1$, as furnished by U.S. Coast and Geodetic Survey, 35,385.5 m

| | Meters |
|---|---|
| $BD=$ | 11.57 |
| $Dc=$ | 8.00 |
| $bc=b_1c=$ | 0.40 |
| $ab=$ | 0.34 |
| $B_1f=$ | 0.11 |
| $fE=$ | 9.30 |

The $D$ of the formula will be

$$D=ab+bc+cD+DB_1+B_1f+2fE=35,424.5\text{ m}.$$

APPENDIX II (JULY 1)

Comparison between C.G.S. pendulum[1] and standard observatory clock:

| Period of Coincidences | Temperature |
|---|---|
| 23m46s | 24°0 |
| 23 44 | 24.1 |
| 23 24 | 24.1 |
| 23 40 | 24.2 |
| 24 00 | 24.4 |
| 1423 | |

$N$ = number of (double) swings per second = 1.00070 at 24°2.

Another comparison gave 1.00074 at 17°.

Whence the temperature correction is − .0000055 per degree.

The clock gains one second per day by comparison with time signals from Washington; giving

$$T=1.00075\text{ at }17°.$$

Another comparison taken August 13 gave the following results:

| Period of Coincidences | Temperature |
|---|---|
| 22m41s | 19°0 |
| 23 15 | 19.2 |
| 23 07 | 19.4 |
| 22 58 | 19.7 |
| 22 50 | 20.1 |
| 23 48 | 21.0 |
| 23 20 | 21.5 |

from which $N=1.00072$ at 19.9°.

[1] The pendulum case was evacuated to a residual pressure of about 1 cm, for which the pressure correction is inappreciable. The amplitude correction was also negligible.

The rate of the standard clock was $+0^s.5$ per day, giving

$$N_2 = 1.000726 \text{ at } 19.9^\circ$$

and

$$N = 1.00074 \text{ at } 17^\circ.$$

## APPENDIX III. MEASUREMENT OF LENGTH OF LINE USED IN DETERMINATION OF VELOCITY OF LIGHT

By WILLIAM BOWIE[1]

As a result of several conferences between Professor A. A. Michelson and the writer on the question of the determination, with high accuracy, of the distance between a point on Mount Wilson and one on San Antonio Peak, California, to be used as a base line in the determination of the velocity of light, Professor Michelson made a request on May 1, 1920, upon the director of the United States Coast and Geodetic Survey, Colonel E. Lester Jones, that the work be done. The director, realizing that the determination of the velocity of light with great accuracy might lead to the determination of distance in terms of the velocity of light and thus might furnish a means of measuring base lines in mountainous regions or on archipelagoes, agreed to measure the distance between the two peaks in question.

Professor Michelson requested that the distance be measured with an accuracy of 1 part in 200,000 but with greater accuracy if possible. In a letter to the director, dated March 23, 1922, Professor Michelson made the statement:

> In my conversations and correspondence with Major William Bowie, I was given to understand that the measurement [of the distance between Mt. Wilson and San Antonio Peak] could be made to an order of accuracy of one in a million which would be highly desirable, if possible, but an order of accuracy of one in 100,000 or 200,000 would be sufficient.

In a letter dated March 14, 1922, Professor George E. Hale, director of the Mount Wilson Observatory, requested the director of the United States Coast and Geodetic Survey to determine by triangulation the distance between Mount Wilson and San Antonio Peak for Professor Michelson's use in his redetermination of the ve-

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locity of light. The director of the Coast and Geodetic Survey replied to Dr. Hale, expressing his deep interest in the work on the velocity of light, and made the statement that the field measurements would be undertaken later in the year.

It was realized that to obtain an accuracy in the determination of the length of a line of triangulation as great as 1 in 200,000 or more would necessitate some modifications and refinements in the methods that are usually employed in what is known as first-order triangulation. It is rather difficult to estimate just what is the accuracy of a line of triangulation carried from a base through a series of triangles or quadrilaterals, but it has generally been supposed that the order of accuracy is about 1 part in 100,000.

Ordinarily, the quadrilaterals used in a chain of triangulation approach the square in shape, that being the most satisfactory shape, if both accuracy and rapid progress are desired. For the Michelson length it was believed best not to depend on a triangulation extended from the Los Angeles base, which had been measured in 1888–1889. There was the possibility that this base had changed its length during the interval since its measurement, and, besides, it was believed that even though the angles of the triangles extending from the Los Angeles base to the Michelson line were remeasured with the greatest care, the lengths carried through the triangles would not have an accuracy of 1 part in 200,000. After careful consideration of the problem, it was decided to measure a base line in the San Gabriel Valley, parallel to the line between Mount Wilson and San Antonio Peak, and to have the base line as close to the foothills as possible in order that the angles involved in transferring the length of the base to the line between the peaks might be of such size as to give the strongest possible connection.

An inspection of the topographic maps of the United States Geological Survey indicated that it was feasible to measure a base of the desired length in this locality. The methods to be employed in the field work were carefully planned at the office of the United States Coast and Geodetic Survey by the members of the Division of Geodesy. The field work was placed in charge of Clem L. Garner, one of the field engineers of the United States Coast and Geodetic Survey who had had a number of years of experience on first-order triangulation and in the measurement of first-order base lines. In his party were a number of engineers of the Survey who had also had experience in the most accurate geodetic work. They were J. S. Bilby, F. W. Hough, and E. O. Heaton.

The computation of the precise levels on the base line was in charge of H. G. Avers. The computations of the measured lengths were made under the direction of W. D. Sutcliffe. Unusual care was employed to take into account all refinements in computation that could affect the final results. The computation and adjustment of the triangulation to carry the length from the bases to the required line extending from the station Michelson on Mount Wilson to the station Antonio on San Antonio Peak were made under the direction of O. S. Adams. Extra computations were made for the deflection of the vertical, and extreme care was employed at all stages of the work. Other mathematicians engaged upon the work, either directly or in consultation, were W. D. Lambert, C. H. Swick, C. A. Mourhess, Sarah Beall, and W. F. Reynolds.

By a field reconnaissance the ends of the base were located where they could be made intervisible by the erection of observing towers of moderate height and where the two peaks at the ends of the Michelson line were visible from each end of the base. It was found, however, that a straight base from end to end could not be measured, owing to the character of the terrain and to the presence of orange groves and other obstructions. Correspondence between the office and the field resulted in the adoption of a plan to measure a broken base, with the various sections located where the measurements could be most easily made.

The methods adopted for the field measurements and the office computations were such as to assure the attainment of an accuracy, for the straight-line distance between Mount Wilson and San Antonio Peak, corresponding to a probable error of about 1 part in 2,000,000, derived from field measurements and observations alone, and to an actual error surely less than 1 part in 300,000. It is the feeling of those who have been engaged in the work that the actual error is somewhere between 1 part in 500,000 and 1 part in 1,000,000.

It was decided to make the distance between the peaks depend on more than one base. This was accomplished by dividing the total base into three parts. The length of each part was obtained during the measurement of the whole base line, as the subsidiary stations were placed along the base itself.

The base was measured in forty-six sections, owing to the character of the terrain, to the presence of obstructions, and to the fact that the base cut the property lines at an angle. Each section of the base was measured at least four times, with 50-m invar tapes which were standardized with extreme accuracy at the Bureau of Standards, Washington, D.C., just before and just after the measurements in the field. The maximum number of measurements of any one section was six. Eight tapes were used in the measurement of the base, and on each section the different measures were made with different tapes.

Along a portion of the line, wooden stakes were set into the ground to support the tapes during the measurements. The length of the tape was transferred by a marker to metal strips nailed to the tops of the stakes at the tape ends. Where the measurement was over a paved highway, special portable tripods made of iron were used. These were set over points which had previously been selected and marked with small iron bolts cemented in drill holes in the pavement. Five supports were used for all the measurements over stakes, and three supports for the measurements over portable tripods and towers. If possible, the intermediate supports were placed in line between the tops of the end supports.

When stakes were used, accurate levels were run over them to determine the difference in elevation of consecutive tape ends. The leveling was done in both directions in order to have a check. Necessarily, where stakes were used, the same grade corrections would be applied to each tape length as measured by each of the four or more tapes. On the other hand, when the movable tripods were used, the distance of the tripod head above the mark on the road was obtained during the measurement with each tape. Later, when levels were run over these parts of the base to give the differences in elevation of the marked points, these differences were combined with the heights of the tripods to give the data from which to compute the corrections for grade for each tape. Necessarily, the grade corrections were slightly different for the different measurements, owing to the fact that the tripods could not be set exactly at the same heights for the different measurements.

At places where the line had to be lifted above orange groves, the tapes were supported on towers. Towers were also used to avoid steep grades where measurements were made across rather sharp depressions in the ground. One of the large uncertainties in base measurements, where the ground is very steep, is in the determination of the grade corrections. In order to avoid this uncertainty, the slope for any tape length was seldom allowed to be greater than 10 per cent.

In computing the base, the lengths of the forty-six sections were projected on to straight lines joining the ends of the three main divisions of the base mentioned above. In general, the difference in direction of a section and the line on which it was projected was only a few degrees, and in only one case did it exceed the limit of 14° specified in the instructions, the deflection angle in this instance being 18°.

The uncertainty in the measured length of a base is due to the following causes: errors in the standardization of the tapes, use of erroneous tension on the tapes during the measurements, effect of wind on the tapes, constant and systematic errors in transferring the ends of the tape to the metal strips on the end supports of the tape, errors in leveling from which the grade corrections are computed, uncertainty in the coefficient of expansion of the tape, and errors in the reduction to sea-level from uncertainty in the average elevation of the base line.

The differences in the lengths of the tapes, as derived from the two standardizations at the Bureau of Standards, one before the measurements in the field and the other shortly afterward, are given in Table I.

The effect of the wind on the tape may be considered as having been negligible, for the measurements were made only when there was a very light wind or during spells of calm weather. The use of five-point supports whenever possible also lessened the effect of the wind on the tape.

The errors in transferring the ends of the tape to the metal strips were undoubtedly small, for the observers doing the marking changed positions between the forward and backward measurements, thus eliminating the systematic effect of any tendency to mark ahead or behind the graduation on the tape. The accidental errors in the marking were very small, owing to the care with which the line on the tape was transferred to the metal strip. The marking was done only when the tape had come to rest after the tension had been applied.

Seven different spring balances were used to apply the tension to the tapes. Each balance was tested twice on every day it was used with a standard 15-kg weight. The balance was placed in a horizontal position during the test, and a small wire led from the tongue of

TABLE I

| Tape Nos. | Differences in Length Millimeters |
|---|---|
| 914 | +0.17 |
| 918 | .18 |
| 921 | .04 |
| 925 | .03 |
| 927 | .19 |
| 928* | ...... |
| 929 | .20 |
| 933 | +0.04 |

* Owing to a misunderstanding when this tape was sent to the Bureau of Standards, its length was not redetermined. The second standardization gave a shorter length for each of the tapes used.

the balance to the weight over a pulley which was practically frictionless. The balances were also tested at the Bureau of Standards after the field work was completed, and the effect of changes in temperature on the springs of the balances was determined. No attempt was made to set the indexes of the balances to make the dials give correct readings, but in the office computations corrections were applied to the measured lengths to account for the errors of the balances as disclosed by the daily tests and for the effects on the springs of any temperature variations.

Extreme care was used in determining the elevation of the tape supports. Levels were run in both directions over the base and, wherever the two runnings failed to agree within a certain amount, a third running was made as a check. The elevations of the ends of the different sections of the base were determined by first-order leveling, and the elevations, as carried through the differences of elevation of the tape ends, were checked with the elevations determined by the first-order levels. It may be assumed, therefore, that the corrections for grade had exceedingly small errors.

The temperature range during the measurements of the base was from 5° to 25° C. The tapes were standardized at 26° and 28° C. The coefficient of expansion of each tape had been determined at the Bureau of Standards with such accuracy that it is believed that any error in the length of the base, due to erroneous temperature corrections, is exceedingly small.

The elevation of the base above sea-level was determined by checked first-order leveling from a nearby bench mark of known elevation, so it is reasonably certain that no error was involved in the sea-level correction.

The agreement of the several measurements of each section indicates that the accidental and systematic errors were largely eliminated by the methods employed. As was mentioned earlier, each section was measured at least four times, each time with a different tape. The average residual of a single measure from the mean of all measurements on a section was 1.1 mm. Only three measures were rejected, and in only thirty-two measures of the remaining total of two hundred eight was the residual more than 2 mm. The average size of the probable error of the sections was 0.5 mm, with no probable errors greater than 1 mm. The maximum probable error of a section was 0.9 mm.

The probable error of the length of the base from field measurements and observations alone was 3.45 mm, or 1 part in 11,600,000. The Bureau of Standards certified that the lengths of the tapes were correct within 1 part in 300,000 and that the probable error did not exceed 1 part in 2,000,000.

During the measurement of the base, observations were made for horizontal directions at four points along the base line, including the two ends, and at San Antonio Peak and Mount Wilson. These observations were made with high-grade direction theodolites. The observations were repeated a number of times in order to eliminate, as far as practicable, the effect of the accidental errors, and they were made on each of several nights in order that any systematic errors resulting from atmospheric conditions might be largely eliminated.

The deflection of the vertical was determined at each of the stations in order that corrections could be applied to the observed directions to eliminate the effect of any tilting of the geoidal surface with respect to the spheroidal surface. The mountain masses to the north of the base line evidently deflect the plumb line to a marked degree. The determination of the astronomic latitudes was made at each of the angle stations except at station Joaquin, which is within a mile of Pasadena west base, and at station Michelson, where it had already been determined. It was assumed that the deflection in the meridian at Joaquin would be approximately the same as at Pasadena west base.

The effect of the deflection of the vertical is to tilt the horizontal plate of the theodolite with respect to the spheroid and, therefore, to cause the measured angles to vary somewhat from the true spherical angles. The astronomic longitude had been determined at Mount Wilson some years before. A comparison of it with the geodetic longitude furnished a deflection of the vertical in the prime vertical at that point, thus making it possible to correct the astronomic azimuth at Mount Wilson for the deflection of the vertical. In the adjustment of the triangulation the true azimuth obtained at Mount Wilson was held fixed. This made it possible to determine the approximate deflection of the vertical in the prime vertical at each of the other stations by a comparison of the astronomic azimuth and the geodetic azimuth at each of them.

The results of the computations showed clearly that it was a wise move to determine the deflections at each of the stations, for the maximum correction to a direction was nearly $1''.5$. A number of directions were only slightly affected, but on the whole the application of corrections for the deflections resulted in appreciably greater accuracy in the determination of the length of the Michelson line.

Owing to the fact that the lines from the base stations to the mountain peaks have inclinations as great as 8° to the horizontal plane, it was necessary to read the stride level of the theodolite whenever pointings were made over the steep lines, in order that corrections might be applied for any tilting of the horizontal axis of the telescope during the observations. This is a refinement in angle measurements that has never been applied in the regular triangulation of the country. In fact, the inclination of a line of triangulation is seldom greater than 1° or 2°, and the error from this source is ordinarily negligible.

The heights of the stations on Mount Wilson and on San Antonio Peak were determined from the base stations by trigonometric leveling. These elevations were required for the purpose of determining the air-line distance between the stations on the two peaks.

The average closing error of the triangles involved in the triangulation was 0.″55. There were only two triangles with closing errors greater than 1″, while the maximum closing error was 1.″57.

The average correction to a direction, resulting from the least-squares adjustment, was 0.″24; and the maximum correction was 0.″83.

The distance, reduced to sea-level, between the triangulation stations Michelson on Mount Wilson and Antonio on San Antonio Peak, resulting from the adjustment, is 35373.21 m. The air-line distance between these two stations is 35385.53 m. No correction was applied to the distance to allow for normal curvature of the line of light between the two stations on account of refraction. A computation was made which showed that this correction is only one part in several million.

The probable error of the straight-line distance from Mount Wilson to San Antonio Peak is 1 part in 6,800,000 from field measurements and observations alone. It is believed that the length of this line has been determined with greater accuracy than that of any other line of triangulation in this or any other country.