An Explanation for Anomalous Oscillations in Terms of Thermal Expansion

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We have reviewed the masters degree thesis by Thomas L. Mahood, California State University, Fullerton reporting experimental determination of average forces due to a transient mass fluctuation.[1] Concurrent with design of equipment to enhance the effect, estimates were sought to improve our measurement confidence. In the process we looked at several alternate classical interpretations of the data that are not ruled out by the tests explained in the subject thesis, copy dated 11/11/99. Two mechanisms in particular that are consistent with the write up and quantifiable are 1) time varying thermal expansions and 2) low frequency oscillations to the piezo stack from the oscillator/amplifier near the pendulum resonance. We have modeled the thermal effects and they appear to explain the movement observed by Mahood and Woodward. A third mechanism, changes in the torsional constant due to heating of the wire, does not appear to be large enough to cause the observed effect, as is correctly indicated in the thesis.

Heating of the piezoelectric stack

The thesis explains the need to monitor temperature and cease operation if the temperature climbs high enough to threaten depoling of the piezo at 350C. A spring thermometer is mounted on the end of the aluminum cylinder away from the piezo stack for monitoring temperature rise. The actual temperature changes that were observed, while alluded to, are not included in the thesis, however the application of power was limited to 5 seconds due to heating, and up to 40 minutes were required to let the temperature drop to a reasonable level for further testing.

Since the indicated temperature rose high enough to delay tests, it must have risen high enough to cause measurable effects such as will be described. The conjecture here is that while the masses in a single piezo unit are balanced on each side of the support arm, the thermal properties are not evenly distributed on each side of the support arm. The result is a time varying thermal expansion of the oscillating unit producing forces and torques that would be difficult to separate from forces and torques that might be produced by the experiment.

A test is described in the thesis done with the units vertical rather than horizontal. This test was done to show that there were no net mechanical motions, and would seem at first to preclude the thermal effect described here as well as other mechanical balance changes that might be possible. However, the effect that we describe is slight and is very much dependent on the lack of damping in the torsional pendulum. In rotating the units, the amount of damping would appear to be many times greater in the vertical plane, and could easily swamp an effect such as above. The different damping is not addressed in the thesis.

Thermal Event Description

During a 5 second run, the piezo stack heats up and then gradually gives up its heat to the aluminum rod. At vacuum levels described, convective cooling is not effective, and most of the heat stays in the unit, appearing as an increase in temperature at the end of the aluminum rod or in the nonresonant case at the end of the brass disk during a 5 second power-on time. The temperature then starts to drop slowly due to radiation after the power is turned off. In 40 to 50 minutes approximately half the heat will be radiated to the walls. The convective contribution to cooling is on the order of 10 times smaller.

During a single 5 second operation most of the electric power driving the piezo unit is absorbed in the piezo stack, causing the temperature to rise. The temperature difference between the piezo and the aluminum or brass end piece allows heat transfer and subsequent temperature rise in the aluminum or brass end pieces along with the resultant thermal expansions.

Since the heat is absorbed during a very short period of time compared to either the equilibrium time or the radiation cooling time, it is very easy to calculate the maximum temperature rise. However, to derive a force term due to the heating, one requires the time history of the heat application and distribution in the unit. The Fourier-Biot equation must be solved to determine the temperature along the unit as a function of time :[2]

$$\mathbf{d^2T/dx^2 + S/k = (l/\mathit{a})\ \partial T/\partial t}$$

where,

- **T** is the temperature,
- **S** is the source term in watts/cm^3,
- **x** is the position along the axis,
- **k** is the thermal conductivity, and
- a is the thermal diffusivity.

The one dimensional simplification shown here is allowed since there is almost no heat loss during the minute and a half total time elapsed, hence there is no radial distribution or end loss. The equation is solved using finite-difference approximation of derivatives. In particular, the central-difference form is used to minimize truncation error, and the differences are selected so that $1/2 \geq a\ \Delta t/(\Delta x)^2$.

With the temperature rise there is thermal expansion. By taking the first derivative of the linear expansion one gets velocity, and taking the second derivative provides acceleration:

$$\mathbf{v = \Delta l/\Delta t = alpha\ x\ L\ x\ \Delta T/\Delta t}$$

or in calculus notation:

$$\mathbf{v = dl/dt = alpha \times L \times dT/dt}$$

By taking the second derivative one gets an acceleration:

$$a = d^2 1/dt^2 = alpha \times L \times d^2T/dt^2$$

Then, since we will assume m is a constant, the usual equations hold for momentum and force,

$$P = mv,$$

and

$$F = dP/dt \simeq ma \text{ (assuming no transient mass changes)},$$

where

m is the mass of the structure connected to the piezo stack,
T is the temperature,
Δ**L** is the distance the piezo stack grows in time Ät,
L is the length of a particular material,
Alpha is the coefficient of linear expansion of the material,
Δ**t** is the length of a single pulse,
v is velocity,
a is acceleration,
P is momentum, and
F is force.

The linear coefficient of expansion is 23E-6 for aluminum, 19E-6 for brass, but 4E-6 for Lead Titanium Zirconate. Similarly, the heat capacity of aluminum is 0.92 J/g-K, brass 0.4 J/g-K, LTZ 0.42 J/g-K. Applying these constants with the dimensions provided in the thesis text, one finds that there is an imbalance resulting in every case. One can predict that the preferred direction of the original pendulum would be CCW, while for the two final arrangements, figure 15 in the thesis text implies a CW direction.

Simplifications

In order to make the calculations described in this report several simplifications were made to the actual experiment as described in Mahood's thesis. First, the unit is assumed to have a constant 1.9-cm diameter, whereas in the resonant unit there are two aluminum disks used to hold the piezo stack together that exceed this diameter. In the non-resonant case, the brass disk is also assumed to have a 1.9-cm diameter. Second, the power delivered to the piezo stack will inevitably have a finite though small rise-time and at the end of the 5 second pulse, a finite decay. Also, the piezo dielectric constant increases with temperature, so the power absorbed in the piezo stack increases during the 5-second power on period. The mass of the bolts holding the assembly together is included in the pendulum calculations, however, the heating of the bolts is not considered in the thermal calculation because they appear to be somewhat isolated by distance and by insulation. We believe that these simplifications either make no difference in the calculation, or reduce the amount of the effect.

Measurement Arrangement

The pendulum is not observed directly, rather, a laser from a stationary source is reflected from a mirror to a viewing scale. Referring to figure 1, the following is an accounting of the movement of the pendulum.

1) ΔX is complete movement = $2 \times 2 \times 3 \times \Delta L \times r/R$
Where there is a factor of

2 for the total movement during oscillation (p 32, theta max),
2 for the laser pointing system (p 30),
3 for the possibility of 3 pulses.

The charts showing the finite difference calculation results are calibrated to the measuring system in that the oscillations shown are those that would appear on the read-out scale described in the thesis and that would be preserved on the video tapes also mentioned in the thesis. A pure harmonic oscillation is shown in each pendulum chart to draw attention to the minimum oscillation that may be occurring below the threshold of observation.

Estimate of temperature rise

Temperature rise due to the power applied to the piezo stack can be estimated assuming that all of the power is converted to heat as the stack is oscillating. The heat energy is the power delivered to the stack times the time of a pulse so that

$$\Delta E = mC_p \Delta T,$$

where m is the mass of the piezo stack originally, but as heat equilibrates, C_p must be evaluated for the entire structure.

The following table provides the thermal constants used in the calculations:

Material	alpha(/^{0}C)	C_pJ/g-^{0}C)	k(w/cm-^{0}C)	*r* (g/cm^3)
PZT	4E-6	0.42	0.018	7.8
Al	23E-6	0.92	2.37	2.7
Brass	19E-6	0.4	1.2	8.5

The PZT thermophysical constants are drawn from Navy Type II measurements equivalent to the EC-65 material type as the EDO company does not publish their own thermophysical values. The following table summarizes the ΔT resulting from the stated power drawn by the piezo stacks, with first value equal to the temperature of the

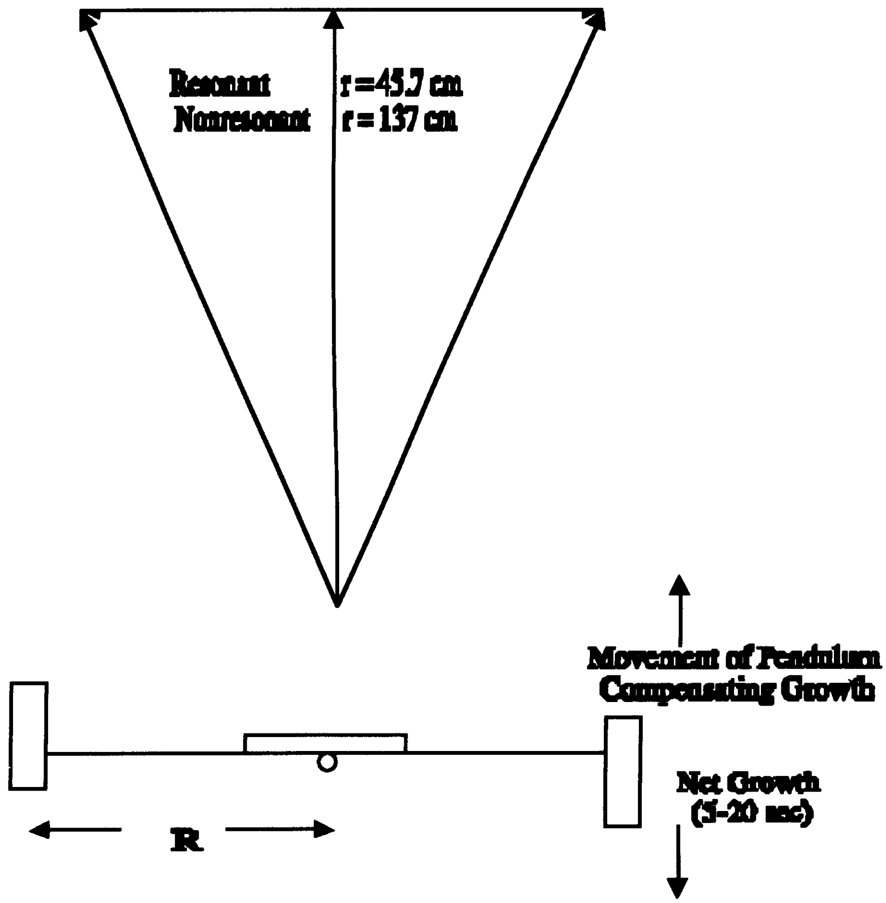


Fig 1 Vertical View of Pendulum

piezo stack with no end pieces and the equilibrium value with end pieces separated by a slash (stack alone/equilibrium):

Experiment	Ave Pwr (w)	ΔE(J)	m(g)	ΔT (1 pulse)
Resonant	81	407	22/60	43/9
Non-resonant (1)	102	510	35/84	35/14
Non-resonant (2)	116	578	35/84	39/16

Resonant Unit Calculations

For the resonant unit, the finite difference grid is set up using 0.1 cm spacing and 0.01 second steps. Three different regions result, (1) the 3.8 cm aluminum cylinder with the aluminum disk joining it to the PZT stack, (2) the 0.76 cm PZT stack, and (3) the aluminum disk cap. Figure 2 shows the unit in schematic form. Figure 3 shows the evolution of temperature across the unit as heat is generated in the PZT and then flows to the aluminum disk on the right and aluminum cylinder on the left. Figure 4 combines the momentum and force as a function of tint. It is interesting to note that the force is strong immediately when heating begins, then tapers off. It is strong again when power is turned off but in the opposite direction. This provides an insight to optimize movement of the pendulum. Figure 5 shows the time history of a pendulum operating with a time dependent force:

$$dx^2/dt^2 + p^2x = F(t)/m$$

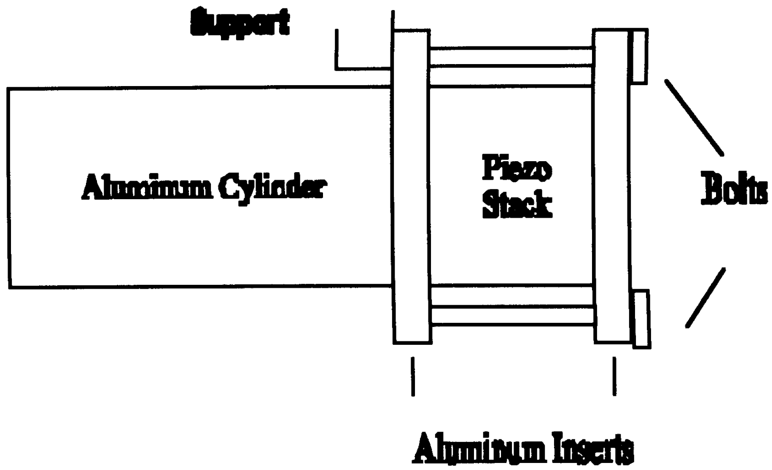
Again, the solution uses the central difference form for the differential to reduce truncation error.

The experimenters explain that oscillations in the pendulum smaller than 0.5 mm total swing cannot be observed. As a result, the pendulum may be at some initial condition other than rest for the first application of power. This can result in various outcomes with regard to the apparent initial direction of the pendulum. Because the force is toward the aluminum cylinder when heating begins and away from the cylinder at the end of the heating cycle, a strategy is suggested to optimize the effect with subsequent pulses once visible motion is apparent.

Non-resonant Unit Calculations

A non-resonant version of the PZT unit is described in the appendix of the Mahood thesis. The stated purpose of this version is to provide a brass disk sized so as to reflect all waves at the boundary with the PZT back into the PZT, thereby concentrating any possible mass changes in the PZT. To examine the thermal excursion of this unit, the finite difference grid is set up exactly like the resonant case, with the unit oriented as it appears to be oriented in the thesis. The schematic for this unit is shown in figure 6.

Fig 2 **Schematic of Resonant Structure**



Left of Support:		**Right of Support:**	
3.8 cm x 23E-6T = 8.74E-5T		**Al Insert**	**0.3 x 23E-6T = 1.38E-5T**
(Al Cylinder)		**Piezo Stack**	**0.76 x 4E-6T = 3E-6T**
		Al Insert	**0.3 x 23E-6T = 1.38E-5T**
Total	**= 8.74E-5T**	**Total**	**= 1.7E-5T**

***The bolts undoubtedly warm up as well, however, it appears that they are isolated by wrappings and the thermal resistance of threads.**

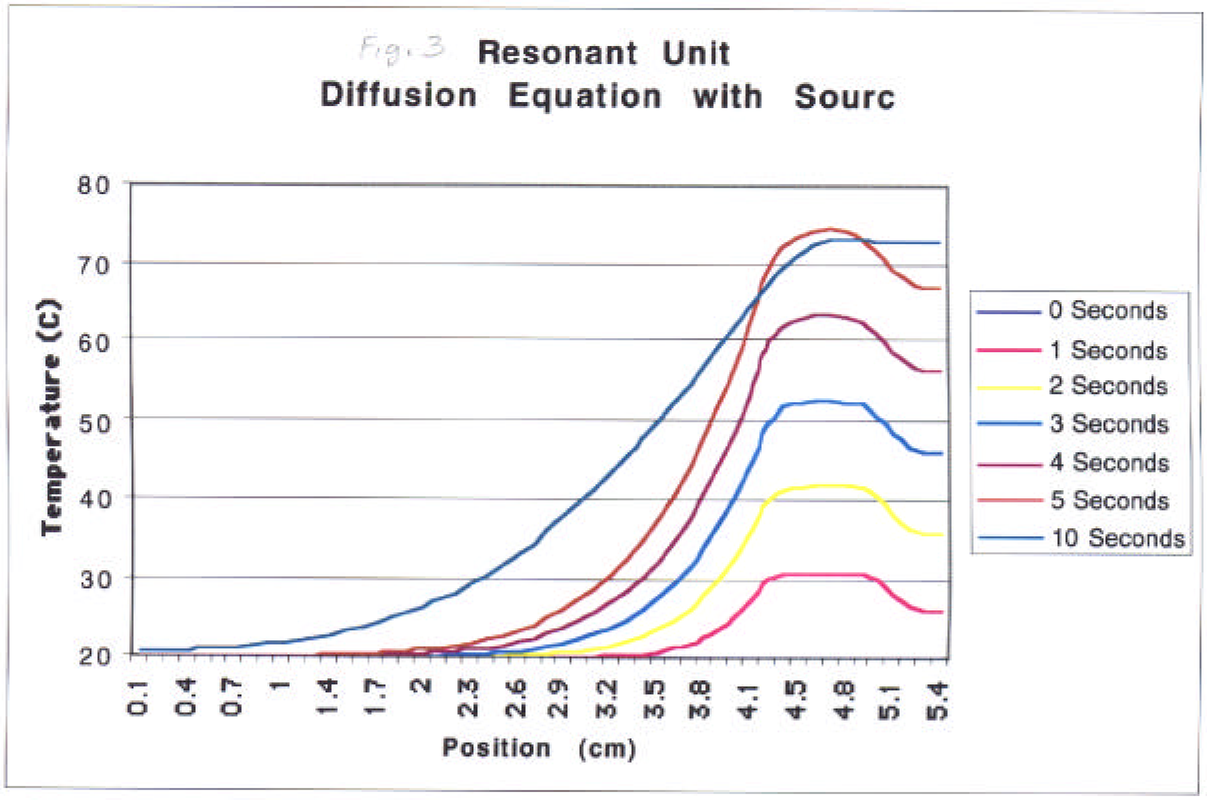


Fig. 3
Resonant Unit
Diffusion Equation with Sourc
Temperature (C)
80
70
60
50
40
30
20
0.1
0.4
0.7
1
1.4
1.7
2
2.3
2.6
2.9
3.2
3.5
3.8
4.1
4.5
4.8
5.1
5.4
Position (cm)
0 Seconds
1 Seconds
2 Seconds
3 Seconds
4 Seconds
5 Seconds
10 Seconds

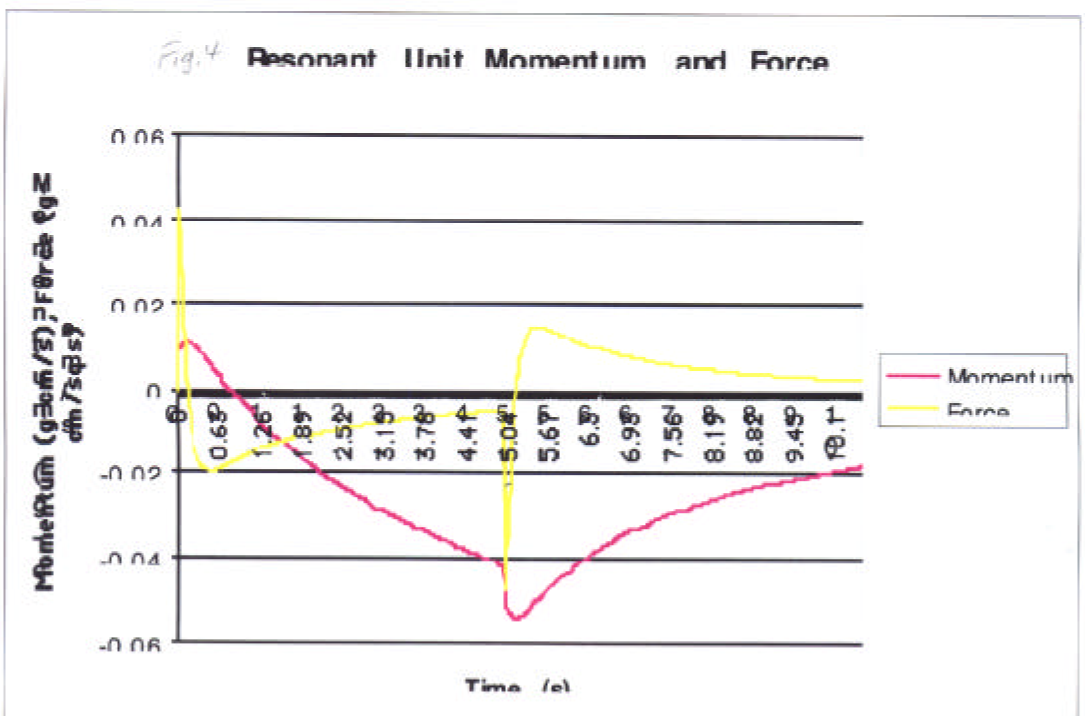


Fig. 4
Resonant Unit Momentum and Force
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
Time (s)
Momentum
Force

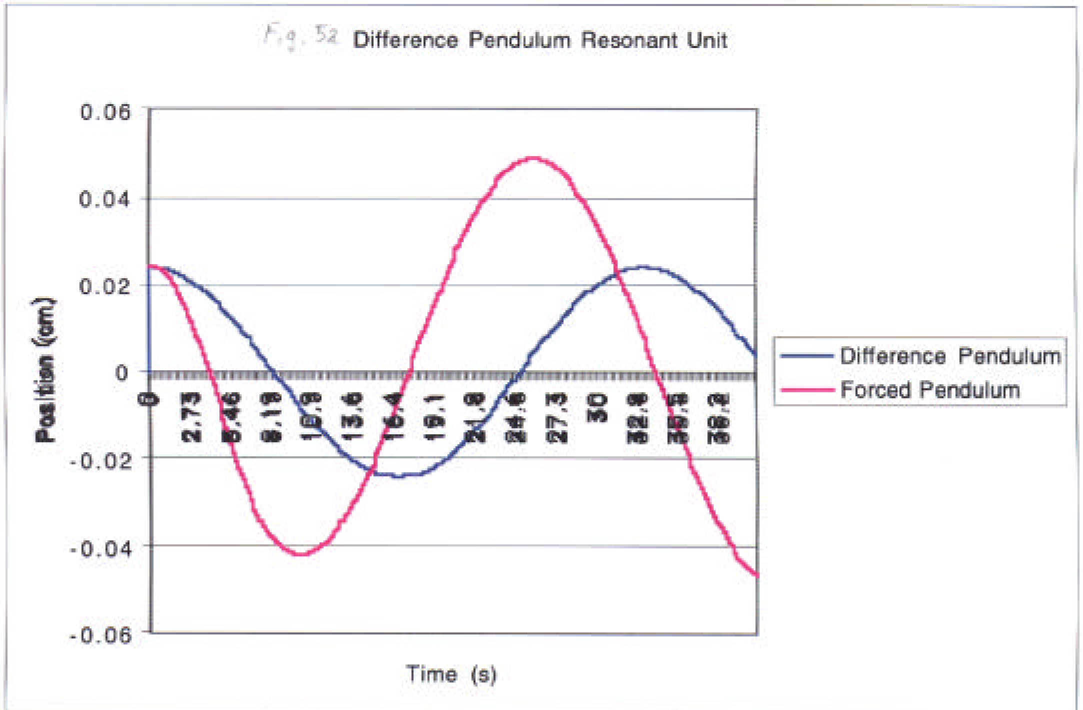


Fig. 5a Difference Pendulum Resonant Unit
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
Position (cm)
Time (s)
Difference Pendulum
Forced Pendulum

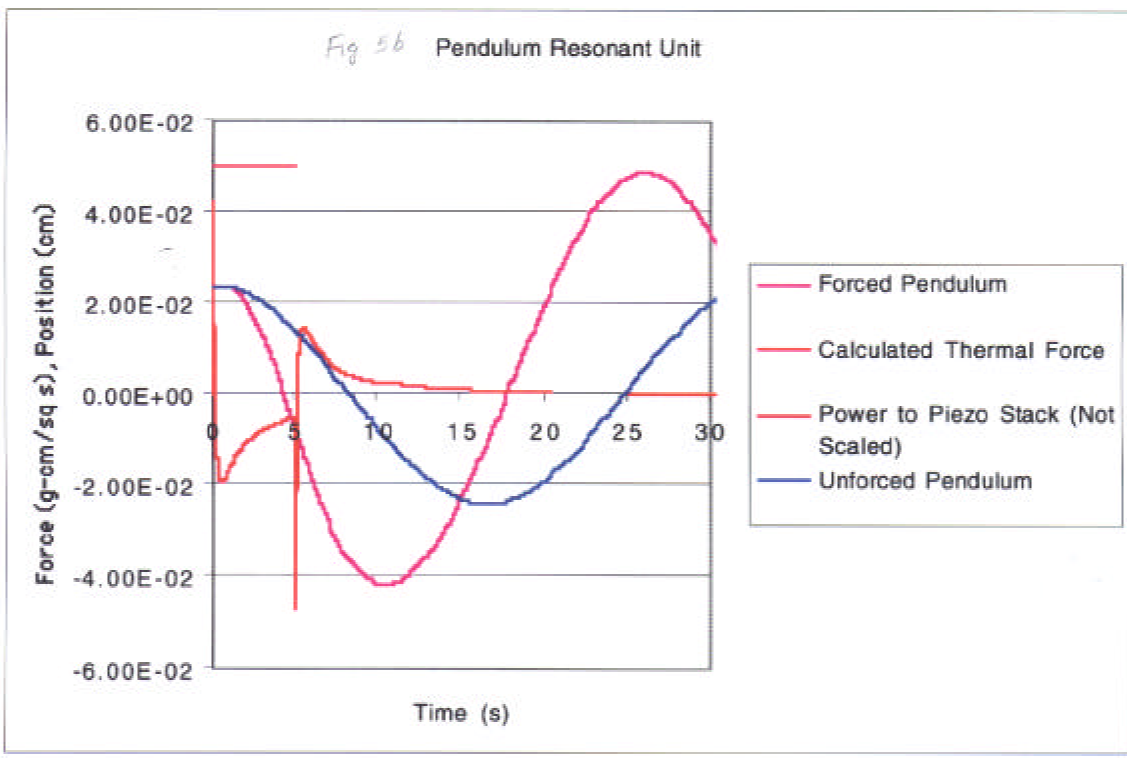
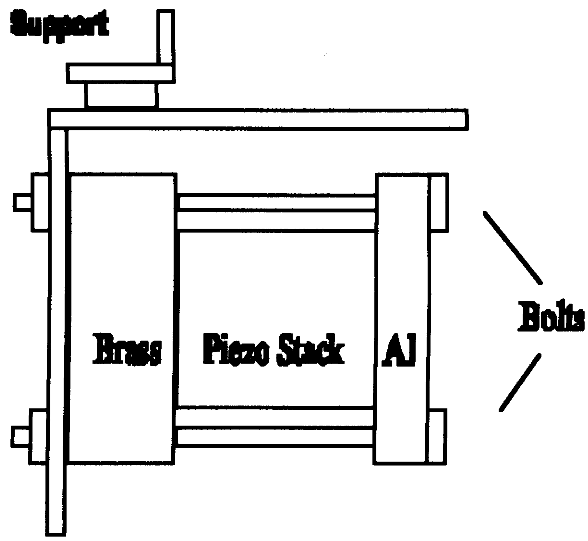


Fig 5b Pendulum Resonant Unit
6.00E-02
4.00E-02
2.00E-02
0.00E+00
-2.00E-02
-4.00E-02
-6.00E-02
Force (g-cm/sq s), Position (cm)
0
5
10
15
20
25
30
Time (s)
Forced Pendulum
Calculated Thermal Force
Power to Piezo Stack (Not Scaled)
Unforced Pendulum

Schematic of Nonresonant Structure



Brass disk	0.9 cm x 19E-6T
Piezo stack	1.6 cm x 4E-6T
Al disk	0.3 cm x 23E-6T
Total	3.2E-5T

***The bolts undoubtedly add to the total expansion in the long run, however, they appear to be isolated by wrappings and thermal resistance in the threads.**

The temperature profile is shown in figure 7, and the temperature effect on momentum and force is shown in figure 8. One can see that the resultant force is larger than in the resonant case, and oriented opposite to it. Part of the increase over the resonant case comes from the change in mounting, the entire unit being mounted to the right of the support bracket in the nonresonant case. The mounting difference results in a change in the shape of the momentum and force. In the model the application of the force due to the first five second pulse complements the motion already present. Likewise, second and third pulses applied at equal times approximately equal to the pendulum period will enhance the motion.

At times the unit will be at a high temperature and cooling off (thus balancing the heating effect) when a test is begun, at other times, such as the first test on a given day, it will be cool and at equilibrium. Any of these conditions will affect the pendulum motion.

Discussion

When one looks at the thesis measurements above the stated 0.5 mm minimum reading into account there appears to be a preference for counterclockwise (CCW) in the case of the resonant unit, and clockwise (CW) in the nonresonant unit. This is most easily understood in terms of the thermal mechanism, while with an electronic mechanism, in the absence of phase control, one would expect random movement to be generated. Further, one could expect that inevitable differences in construction details such as relative efficiency, epoxy thickness or uniformity, torque on bolts, would make the two units on opposite sides of the pendulum behave differently.

Although it isn’t mentioned in the thesis, one could imagine a desire to switch units, running unit 2 while unit 1 cooled, and vice versa to allow faster data acquisition. This would result in a balancing situation - as one unit cooled the other would at first heat up in the piezo, adding to the effect caused by the other one cooling. Later in the run, the two effects would oppose one another as the heat flows to the aluminum cylinder. Since the rate of cooling will depend on the absolute temperature of the unit, a really hot unit will be cooling faster than a moderately warm unit, and thus will provide more movement counter to the expected force.

For the second design, the effect will be larger and more easily observed, in that heating always causes motion away from the stack, while cooling causes motion toward the stack. This is clearly indicated in the runs S1-S12.

Conclusions

A mechanism that does not require new physics appears to be large enough to explain the experiment. Further evaluation could be accomplished in two ways. First, with a knowledge of temperatures and temperature rise during the experiment, the estimates shown above could be refined. Second, with more information about the system described in the breakthroughs in physics paper, an evaluation could be made to see if the same physics principles apply there.

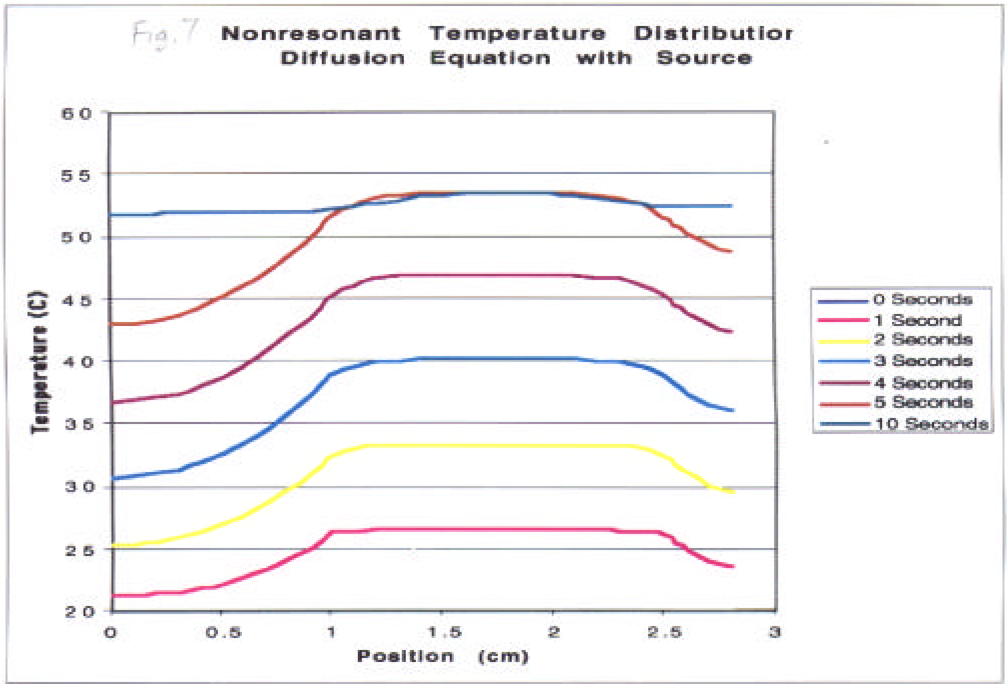


Fig. 7 Nonresonant Temperature Distributior
Diffusion Equation with Source
Temperature (C)
60
55
50
45
40
35
30
25
20
0
0.5
1
1.5
2
2.5
3
Position (cm)
0 Seconds
1 Second
2 Seconds
3 Seconds
4 Seconds
5 Seconds
10 Seconds

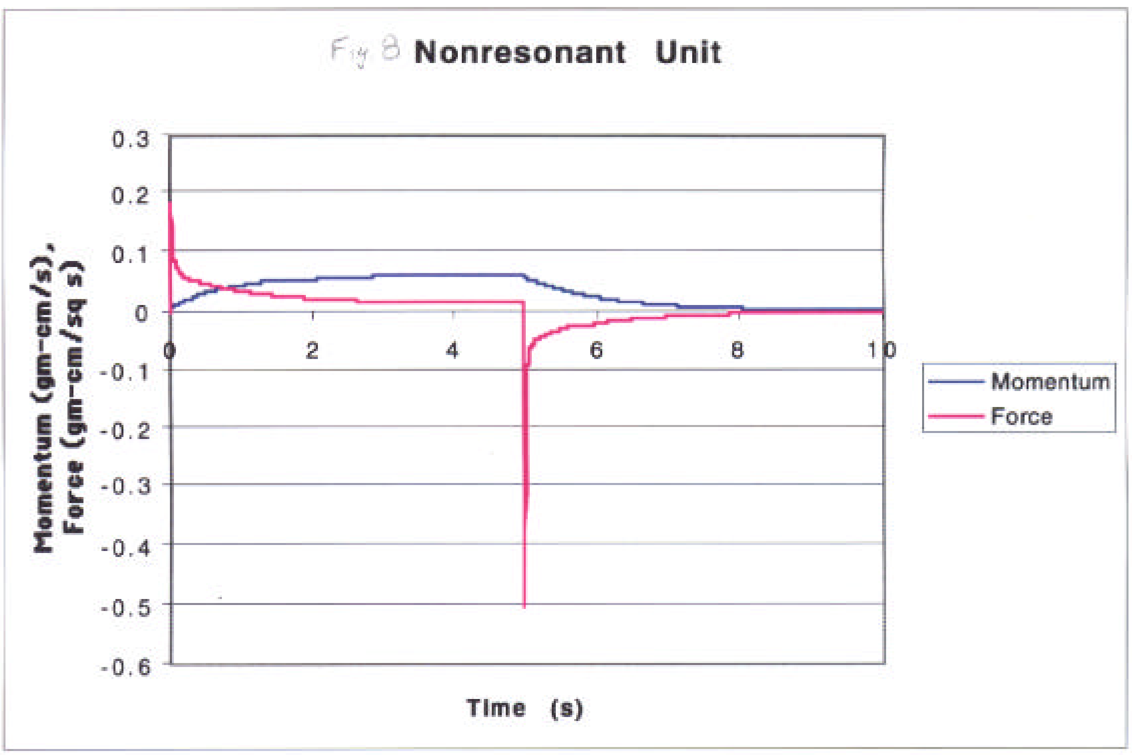


Fig 8 Nonresonant Unit
Momentum (gm-cm/s), Force (gm-cm/sq s)
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
0
2
4
6
8
10
Time (s)
Momentum
Force

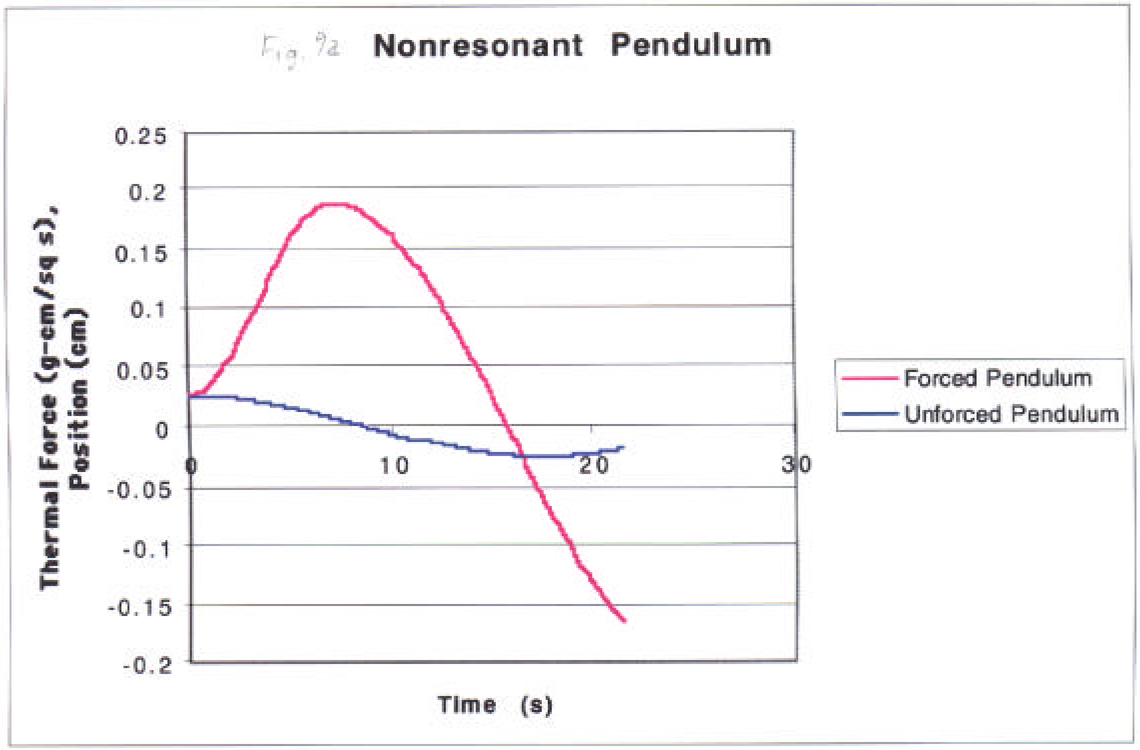


Fig. 9a
Nonresonant Pendulum
Thermal Force (g-cm/sq s), Position (cm)
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
0
10
20
30
Time (s)
Forced Pendulum
Unforced Pendulum

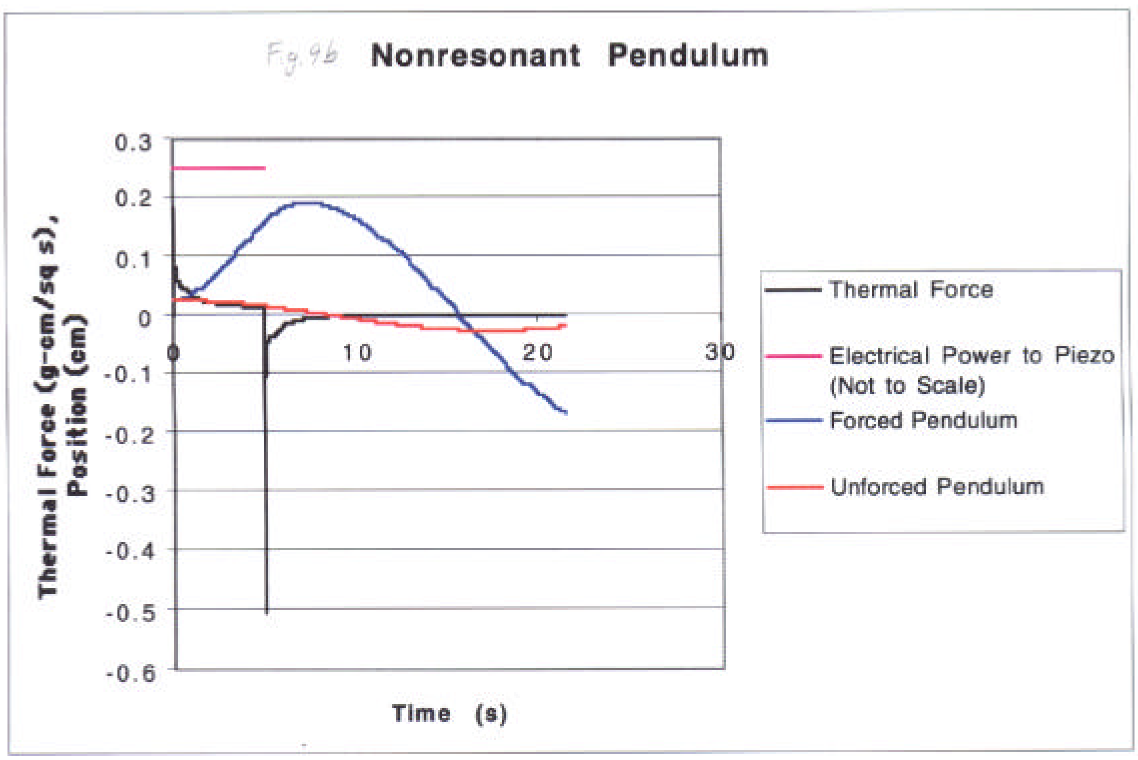


Fig. 9b
Nonresonant Pendulum
Thermal Force (g-cm/sq s), Position (cm)
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
0
10
20
30
Time (s)
Thermal Force
Electrical Power to Piezo (Not to Scale)
Forced Pendulum
Unforced Pendulum

References

T. L. Mahood, "A Torsion Pendulum Investigation of Transient Machian Effects," California State University Masters Thesis, November, (1999).

S. Kakac, Y. Yener, Heat Conduction , Taylor and Francis, (1993).